STUDENT INVESTIGATION BOOKLET

Mathematical Mysteries

Join Magda to simplify maps, trace magic paths, defend networks and justify why your solution must work.

Map maker: ______________________________

Bring: pencil, eraser, ruler and at least four coloured pencils. If colours are hard to distinguish, use dots, stripes, crosses and waves as patterns.

Magda's Network Words

A network keeps the connections and removes detail that does not matter.

Sketch one symbol for each word as it appears in the online story.

VERTEX

A place or point

One vertex; two or more vertices.

EDGE

A connection

A road, bridge or link between two vertices.

DEGREE

Edges meeting here

The degree of a vertex is the number of edges that meet it.

ROUTE OR TRAIL

A journey through edges

A one-stroke trail uses every edge exactly once.

CONNECTED

Every place can be reached

A network is connected when you can travel along edges from any vertex to every other vertex.

FC

HIDDEN FIGURE SPOTLIGHT Fan Chung

Fan Chung finds patterns in very large networks.

Try her question: Which connection seems most important?

Source: University of California San Diego

1 Simplify an everyday network

Choose a school, park, transport or family network.

Vertices could represent ____________________

Edges could represent ____________________

Places Become Points

Connection matters more than the position of a dot.

1 Copy the connections

Use the digital Network Builder. Draw your network here with a different layout, but keep every connection the same.

2 Record the degrees

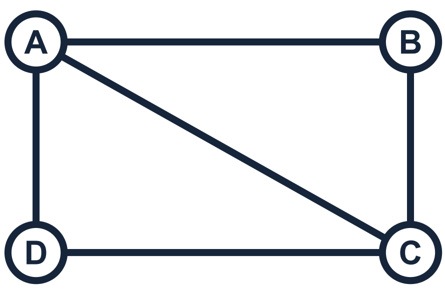
Label the vertices A, B, C and D.

Vertex	Degree	Odd or even?
A		
B		
C		
D		

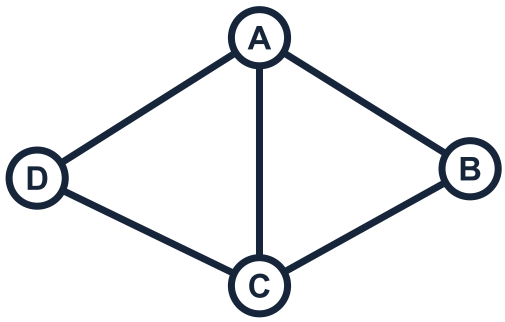
Total number of edges: ______

3 Same network or different network?

Look at these diagrams. Decide whether the connections are the same.



Network X



Network Y

○ Same connections ○ Different connections

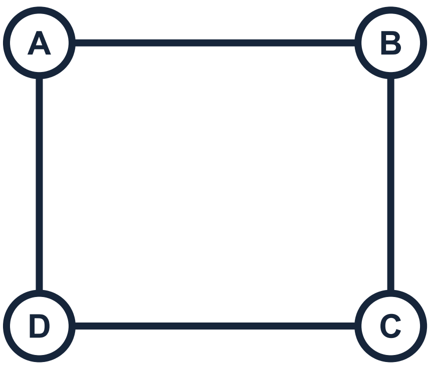
I know because ______________________________

Predict → Test → Notice

Trace every edge exactly once. Do not lift your pencil or retrace an edge.

First mark a predicted start. Then trace. If you get trapped, keep the attempt visible and record where it happened—failed routes contain useful evidence.

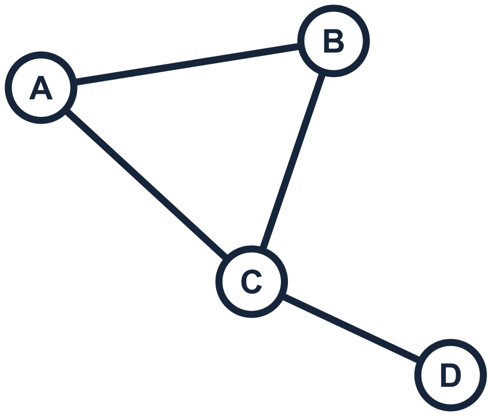
MYSTERY A



Predicted start: _____ Finish: _____

Possible? ☐ Yes ☐ No

MYSTERY B



Predicted start: _____ Finish: _____

Possible? ☐ Yes ☐ No

Predict
Where will I start?

Test
Where did I travel?

Notice
Where could I get trapped?

Revise
What will I change?

1 Count before you trace again

Network	Vertices with even degree	Vertices with odd degree	Successful start(s)
Mystery A			
Mystery B			

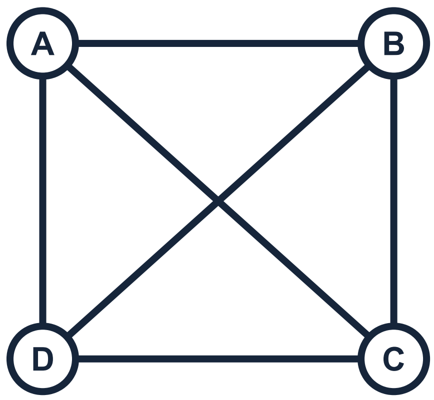
I think the start matters when ______________________________

A degree is odd when it cannot be split into pairs. Circle odd-degree vertices after—not before—your first prediction.

Can Every Network Work?

Use degree evidence to predict before tracing.

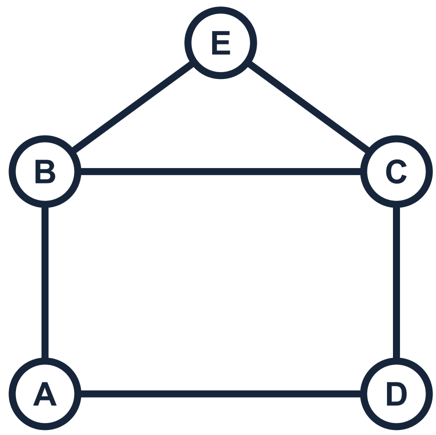
MYSTERY C



Odd vertices: ____________________

Prediction: □ possible □ impossible

MYSTERY D



Odd vertices: ____________________

Prediction: □ possible □ impossible

1 Test only after predicting

Trace both. If a network is impossible, write why. If it is possible, record the starts that can work.

MYSTERY C

____

____

MYSTERY D

____

____

My rule idea: A connected network can have a one-stroke trail when the number of odd-degree vertices is ________________. If there are two odd vertices, the route must start and finish _________________________.

2 Use the degree clue

Mystery C cannot work because it has ____________________ odd vertices.

Mystery D must start at ____________________ or ____________________ because these are its odd vertices.

Connected matters: a network split into separate pieces cannot be covered by one continuous trail, even if all degrees are even.

The Seven Bridges Mystery

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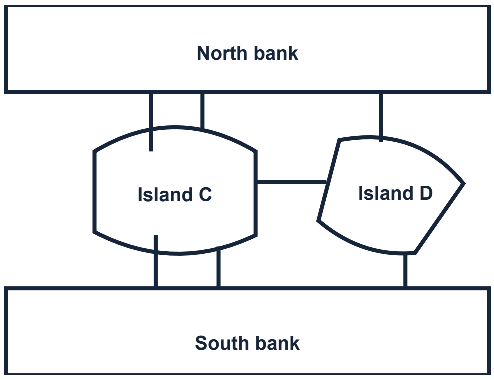
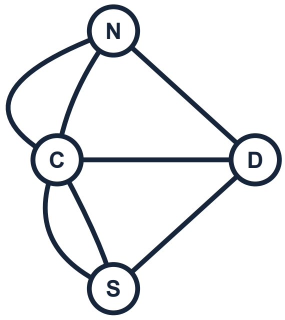
Keep the connections; remove the distracting map detail.

Can one route cross all seven bridges exactly once? Turn each land region into a vertex and keep each bridge as an edge.

MAP MODEL



NETWORK MODEL



Land-region vertex	N	S	C	D
Degree				
Odd or even?				

1 Make the decision before tracing

○ A one-stroke bridge route is possible ○ It is impossible

My degree evidence is ______________________________

2 What stayed the same?

The pictures look different, but every land region and bridge still has a matching point or edge.

The route problem stays the same because

____

HIDDEN FIGURE SPOTLIGHT

LE

Leonhard Euler

In 1736, Euler realised that the bridge puzzle could be solved by studying connections instead of distances. His idea helped begin graph theory.

Source: Mathematical Association of America

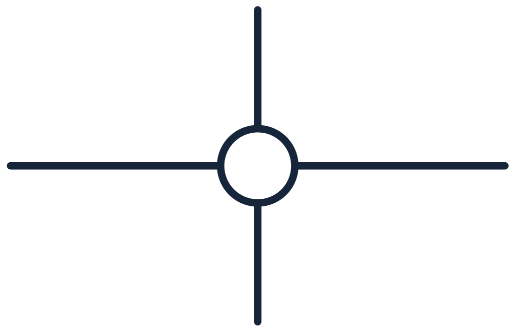
Odd, Even and Why

Organise the evidence from several networks before stating a rule.

Network	Number of odd-degree vertices	One-stroke trail?	Where can it start and finish?
Mystery A			
Mystery B			
Mystery C			
Mystery D			
Seven Bridges			

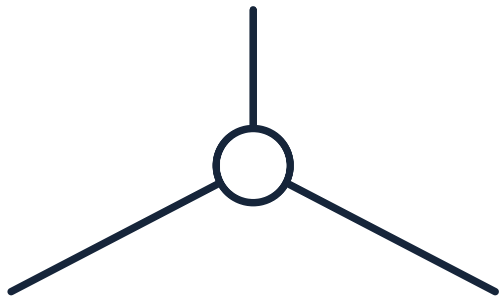
Why pairs matter

At a vertex in the middle of a trail, one edge brings you in and another edge takes you out. Draw pairs around this degree-4 vertex.



Why odd vertices are special

At an odd vertex, one edge is left unpaired. That unpaired edge can belong at a route's start or finish. Draw pairs around this degree-3 vertex.



Complete the rule. In a connected network, a one-stroke trail is possible when there are _____ **or** _____ odd-degree vertices. With no odd vertices, a trail can finish where it starts. With two odd vertices, it must start at one odd vertex and finish at the other.

Why does the rule work?

The pairing idea works because ____

____

____

A useful rule does more than describe successful examples: it explains why a route must start, finish or fail.

Protect the Network

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Predict what becomes unreachable before removing a connection.

An **important edge** is a connection whose removal splits the network. An **important vertex** is a place whose removal splits the remaining network.

TOWN NETWORK

1 Edge attack

Circle one edge whose removal would split the town into two groups.

I chose edge _____ because:

____

____

____

2 Vertex attack

Cross out one vertex whose removal would isolate at least two other vertices from each other.

I chose vertex _____ because:

____

____

____

3 Make the smallest repair

Add exactly one new edge to the town network so that removing edge B–D no longer splits it. Label your edge, then explain the alternate route.

____

____

Before my new edge	After my new edge
Places reachable from A if B–D fails: _______________	Places reachable from A if B–D fails: _______________

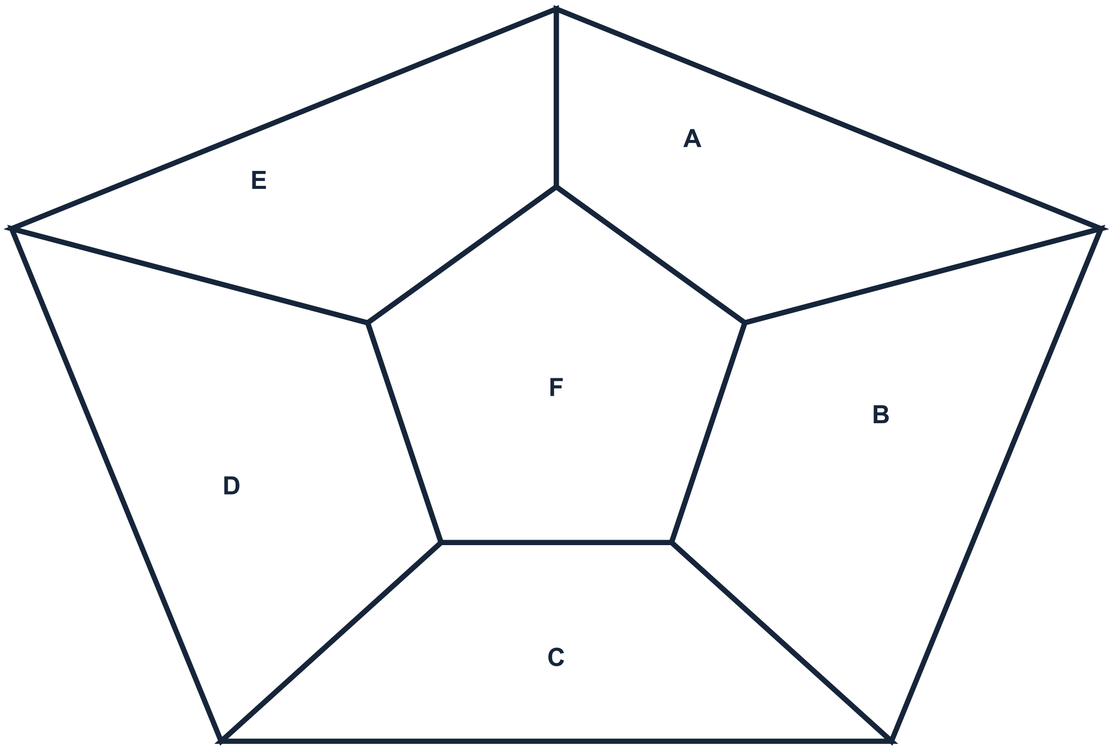
Real systems such as transport, internet routing and supply networks use alternate connections to reduce the effect of one failure.

Colour the Map

Regions that share a boundary must differ. Touching at one point does not count.

Use no more than four colours. If you prefer patterns, use blank, dots, stripes and crosses. Write a colour or pattern key before you begin.

1 = ______________ 2 = ______________ 3 = ______________ 4 = ______________



1 Check

Tick each shared boundary after the regions on its two sides have different colours or patterns.

I found _____ shared boundaries.

2 Find the smallest number

Colour ring A–E first. The smallest number that works for the ring is _____.

F touches all five. Can F reuse a ring colour? □ Yes □ No

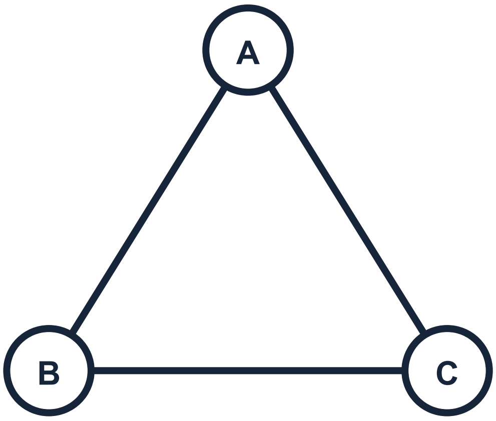
Smallest number for the whole map:

_____

How Many Do We Really Need?

Using three colours does not yet show that two colours are impossible.

NEIGHBOUR CLUE



Each line means the two regions share a boundary.

1 Show that three are needed

A touches B. B touches C. C also touches A.

Could all three use one colour? □ Yes □ No

Could all three use only two colours? □ Yes □ No

Smallest number that works: _____

Explain your answer

A, B and C need three colours because every region

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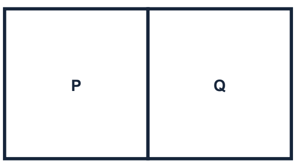
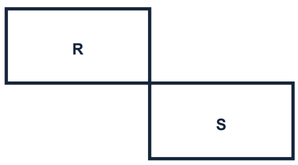
2 Point or boundary?

For each pair, decide whether the regions are neighbours under the map-colouring rule.



P and Q: □ neighbours □ not neighbours



R and S: □ neighbours □ not neighbours

Accuracy check: Today you validate examples and justify why a particular group needs a certain number of colours. You are not proving the full Four-Colour Theorem.

Use a thick pencil mark along a shared boundary if the difference between point-touch and edge-touch is hard to see.

Create a Mathematical Mystery

Design it, solve it yourself, then invite someone else to test it.

My mystery type: ○ Magic Path ○ Network Attack ○ Map Colouring

PUZZLE FOR THE SOLVER

PRIVATE SOLUTION / ROUTE

1 Write precise rules

The solver must ______________________________

2 Show why your answer works

My answer works because ______________________________

Creator check

- ☐ I solved my own puzzle.
- ☐ The rules allow one clear decision.
- ☐ My explanation uses a property.

Tester note

What was clear? ____________

What needs revision? ____________

Tester initials: ______

A tester finding an ambiguity is valuable evidence. Improve one rule or one connection before sharing your final mystery.

Think Like a Mathematician

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Use the evidence in your booklet to make your final claims.

G1 · Plan ahead	I considered how one vertex choice affected later steps.	□ Ready □ Revisit
G2 · Connections	I represented a situation with vertices and edges.	□ Ready □ Revisit
G3 · Justify	I used properties and evidence to explain why.	□ Ready □ Revisit

1 A choice that changed the whole network

Describe one start, edge, vertex or colour choice and its later consequence.

2 Maths of connections

Where might a person use network mathematics outside this workshop?

○ GPS or transport ○ internet routing ○ social networks ○ another system

3 Final justification

I claim ______________________________

My evidence is ______________________________

Therefore ______________________________

One question I still have:

____